

# Quadrature Amplitude Modulation : A simulation study

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November 29, 2005

## **Abstract**

This report details our implementation and observations of a 16-QAM and 4-QAM system. The modem was implemented in software using Simulink. Error bit rates as a function of SNR were studied using Monte Carlo estimation techniques.

# 1 Introduction

Quadrature Amplitude Modulation, QAM, has fast become the dominant modulation mechanism for high speed digital signals. From the wireless 802.11 protocols to ADSL modems to personal communicators for the military, QAM has become a necessary part of our daily lives. With increases in processing power, QAM as a part of software defined radio (SDR) schema is now easily achievable. This report details a Simulink implementation of a 4 and 16-QAM communication system which would be suitable for a SDR system.

## 2 QAM background

Quadrature Amplitude Modulation (QAM) is a modulation scheme which is carried out by changing (modulating) the amplitude of two carrier waves. The carrier waves are out of phase by 90 degrees, and are called quadrature carriers - hence the name of the scheme.

### 2.1 Constellation diagrams

A constellation diagram is the representation of a digital modulation scheme on the complex plane. The diagram is formed by choosing a set of complex numbers to represent modulation symbols. These points are usually ordered by the gray code sequence. Gray codes are binary sequences where two successive values differ in only one digit. The use of gray codes helps reduce the bit errors. The real and imaginary axes are often called the in-phase and the quadrature. These points are usually arranged in a rectangular grid in QAM, though other arrangements are possible. The number of points in the grid is usually a power of two because in digital communications the data is binary.

Upon reception of the signal, the demodulator examines the received symbol and chooses the closest constellation point based on Euclidean distance. It is possible to transmit more bits per symbols by using a higher-order

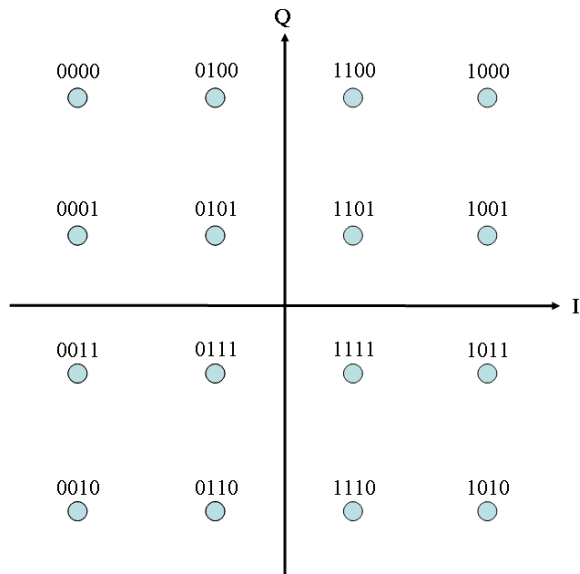


Figure 1: 16-QAM constellation diagram

constellation QAM, but this is more susceptible to noise because the points are closer together, resulting in a higher bit error rate (BER).

Figure 1 shows a 16-QAM constellation diagram.

### 3 Implementation

The goal of our implementation was to mimic an ideal DSP based QAM communication system. As software defined radio becomes more and more prevalent, the only analog stages in communication systems will be the RF transmission and reception stages. After investigating several alternatives, we decided to implement our QAM communication system in Simulink. In the interest of brevity, only the implementation of the 16-QAM system will be discussed in detail. The 4-QAM implementation is easily obtained from the 16-QAM implementation by simplifying the lookup tables for digital to analog and analog to digital conversion. Figure 2 shows a high level overview of the communication system.

The implementation can be broken down into three main stages: modulation,

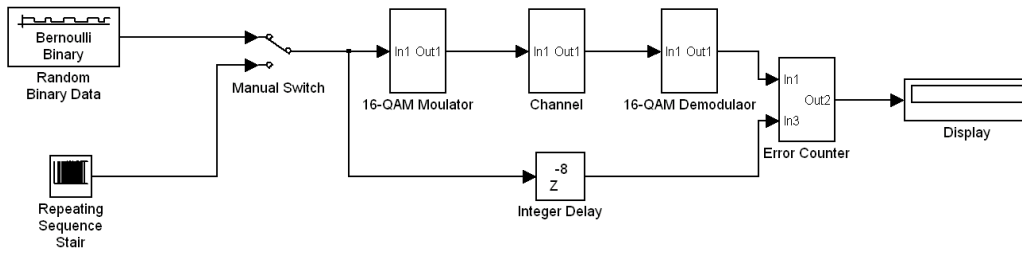


Figure 2: The 16-QAM complete modem block diagram

transmission and demodulation. The modulation and demodulation stages are comprised of several distinct subsystems which will each be discussed in detail below.

### 3.1 Modulation

Figure 3 shows an overview of the modulation block. Each of the subsystems within the block will be explained below.

#### 3.1.1 Serial to Parallel Conversion

The system must be able to modulate a serial digital input as most devices want to output data in this manner. The first stage in the modulation block

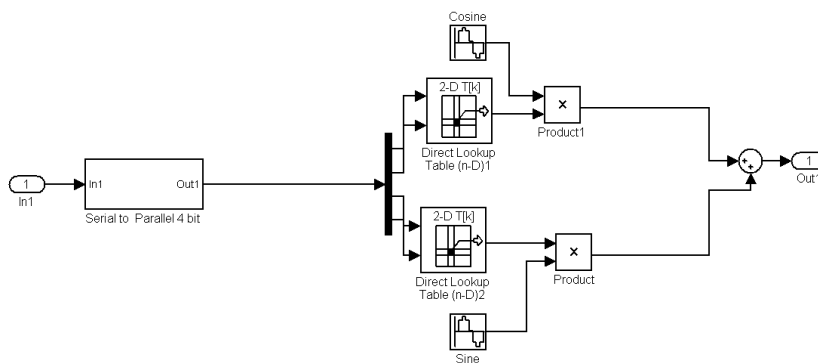


Figure 3: The 16-QAM modulation block diagram

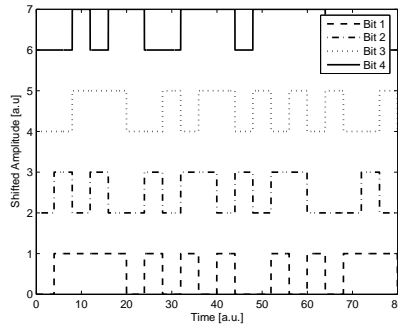


Figure 4: Parallel Signals for the 16-QAM system

need to be a serial to parallel conversion to change the bit stream into  $\log_2 M$  streams, where  $M$  is number of symbols in the constellation. The bit rate of each of the new streams is only  $1/\log_2 M$ .

### 3.1.2 Digital to Analog Conversion

In the case of 16-QAM there are then 4 input streams which index a lookup table. One of the simpler methods implementing the lookup table for gray coded 16-QAM uses two two-bit lookup. One of the signals then modulates the quadrature,  $Q$ , carrier and the other modulates the in-phase,  $I$ , carrier [5]. As mentioned earlier, square 16-QAM uses two amplitudes and four phases to modulate the carrier. The 4-QAM implementation uses one amplitude and four phases.

For our simulated communication system the system can never become truly analog. If the sampling rate of the carrier is sufficiently greater than the symbol rate, the simulated system should approximate an analog system well. After some experimentation, a carrier sampling rate of  $200 f_{sym}$  was determined to be a good trade-off between memory usage and accuracy.

The final stage of the modulation is simply the addition of the  $Q$  and  $I$  signals to form the final M-QAM modulated signal

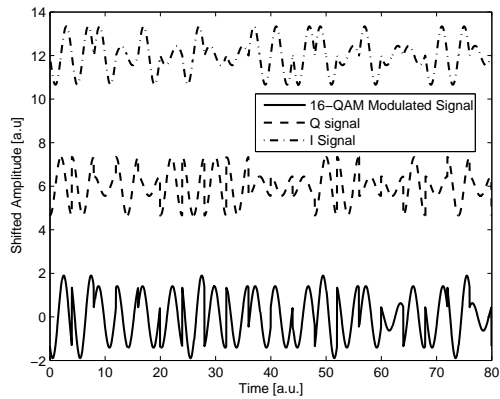


Figure 5: Modulated Signals for the 16-QAM system

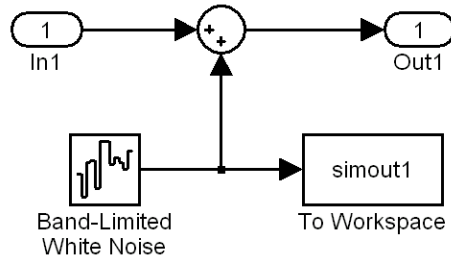


Figure 6: The channel block diagram

## 3.2 Transmission

The transmission channel has several effects on the signal in a physical system. The finite bandwidth of the channel will cause distortion in the ideal signal, noise from various sources will also be introduced and finally the channel may also attenuate the input signal. Other effects, such as as dispersion, may also be present, but we will not concern ourselves with them here. Figure 6 shows the model we implemented for the channel.

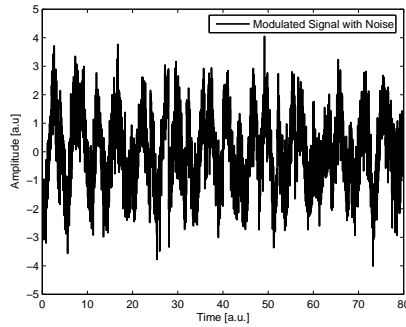


Figure 7: Modulated Signal after Noise is added

Each of the effects mentioned above will effect the BER of the received signal. The effects of attenuation and noise can be modelled by adding gaussian white noise to the signal, effectively changing the signal to noise ratio present at the receiver. The effects of finite bandwidth can be modelled by a simple filter. Both of these effects can be studied in the simulated system we created.

### 3.3 Demodulation

The demodulation section is by far the most complicated part of the QAM model. The demodulator must detect the phase and amplitude of the signal, decode the symbol based on the phase and amplitude and then finally convert the data back to a serial stream. In order to complete the symbol demodulation, recovery of both the carrier and the symbol clock is required. Figure 8 shows an overview of the demodulation block. Each of the subsystems within the block (Figure 9) will be explained below.

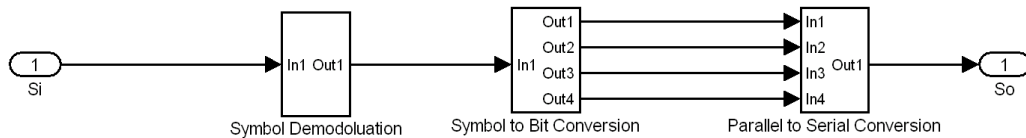


Figure 8: The 16-QAM demodulation block diagram

### 3.3.1 Carrier Recovery

Carrier recovery via squaring does not work for QAM systems as power in  $I$  and  $Q$  signals tends to be the same on average so a cancellation occurs [2]. Instead four-times carrier recovery is often used. Rustako [4] *et al.* have shown that four-times carrier recovery is more effective than any other signal multiplication schemes. Other decision based carrier recovery schemes exist, but they are quite complicated to implement [2].

The first step in the four-times carrier recovery is a multiplication of the signal by itself four-times. From here we use the standard coherent demodulation technique with a PLL tuned to  $f_c$ . The feedback of the PLL contains another four-times multiplier so that the loop locks onto  $4f_c$  at the input but the output frequency is  $f_c$ .

### 3.3.2 Clock Recovery

In any QAM scheme clock recovery poses a significant problem for the recovery of a quality signal. A wide variety of methods are available for clock recovery, however, their usefulness actually depends on the type of transmitted data [2]. The simulation software will be using a Bernoulli coded random bit sequence so none of the decision based schemes would have an advantage. In this case the best choice would be a synchroniser clock recovery circuit. To reduce complexity, a simple times-two clock recovery system was implemented. After a significant amount of tuning, the clock recovery still

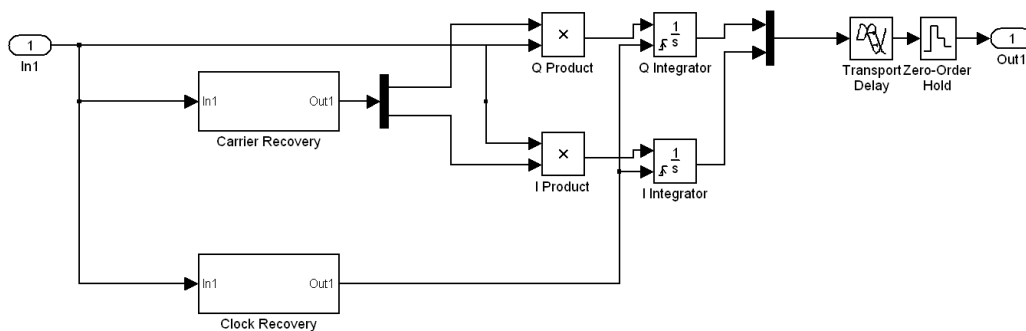


Figure 9: The 16-QAM symbol demodulation block diagram

had a significant amount of jitter. As expected, the jitter was dependent on the data being transmitted. Hanzo [2] recommends decision directed carrier recovery as the best recovery scheme for random signals.

### **3.3.3 Symbol Demodulation**

With both a carrier and clock recovered, the symbol can now be determined from the incoming signal. The heart of the demodulator is a time limited integration. First the signals are split and multiplied by  $\sin(2\pi f_o t)$  and  $\cos(2\pi f_o t)$ . The clock is then used to reset an integrator that acts on each of these input signals. The two analog signals must then be sampled and held just before the end of each period. This process is implemented with an appropriately timed zero-order hold in simulink.

### **3.3.4 Symbol to Bit Conversion**

The symbol to bit conversion requires two stages: a quantizing stage and a decision stage. The quantizer was implemented by the quantizer block in simulink and set to integer levels to reduce problems in the decision stage. The decision stage was implemented with a simple lookup table which outputs individual bits based on the two quantized levels detected.

### **3.3.5 Parallel to Serial Conversion**

The final stage in the implementation must undo the first stage. A switch is used to time-division multiplex the four individual bit signals into a single sequence.

## **3.4 Error Counter**

Figure 11 shows the block diagram we implemented for the error counter. This block uses the XOR operation to keep a count of demodulated bits that don't match the input.

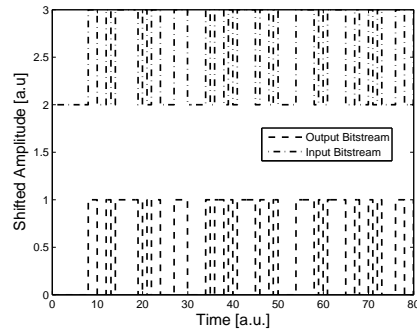


Figure 10: Serial input and Output signals after delay compensation

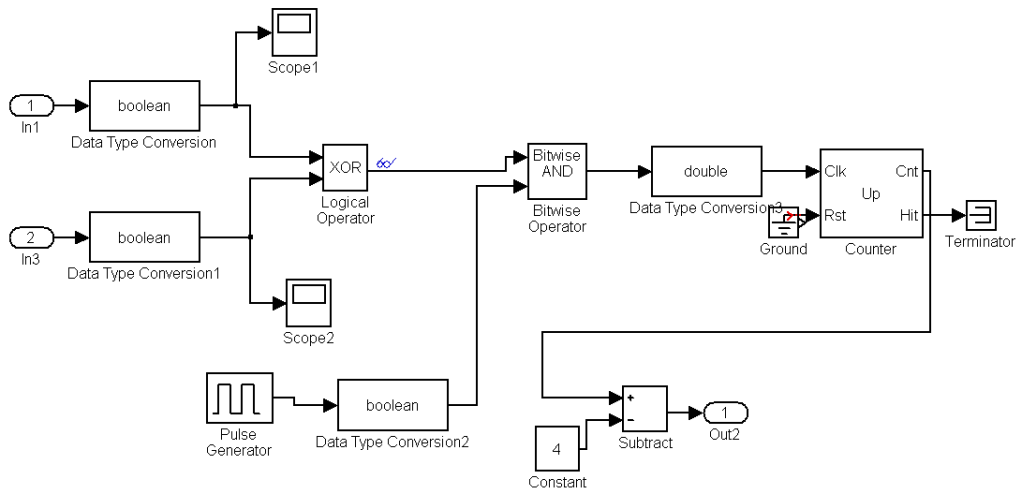


Figure 11: The 16-QAM error counter

## 4 Analysis

Most real communication systems are far too complex to allow for closed form analysis. With the increasing speed of processors, large scale Monte-Carlo analyses provide a mechanism to quantify the operation of a communications system. For examination of a communication system, two random processes are used. First, the transmitted data is randomized so each individual bit is independent of the other bits. Then, random noise is added within the channel.

Using the implementation described in section 3, a Monte-Carlo simulation can be performed by sending a large numbers of bits. By calculating the number of experiments, which are the bits, that are decoded incorrectly, an estimate of the bit error rate (BER) is obtained. By the central limit theorem, this estimated BER will converge to the actual BER as  $1/\sqrt{N}$  for large  $N$ [3].

In the following analysis,  $10^5$  experiments were performed at each value of the SNR. Larger numbers of iterations led to memory problems in simulink. As analytical expressions for the BER as a function of the SNR are only available for perfect clock and carrier recovery, these circuits were replaced by ideal generators for the process of this simulation. In a real system, the effects of jitter and mismatching would increase the BER.

### 4.1 16 QAM Analysis

As mentioned above, several sources have derived analytical expressions for the Bit Error Rate as a function of the SNR per bit for various square QAM implementations. Equation 4.1 was derived by Hanzo et al. and gives the probability of a bit error as a function of  $\gamma = \frac{4}{5}SNR$  [2].

$$P_E^{16} = \frac{1}{4} [3Q(\sqrt{\gamma}) + 2Q(3\sqrt{\gamma}) - Q(5\sqrt{\gamma})] \quad (4.1)$$

Equation 4.1 can now be compared the Monte Carlo results as shown below in figure 12. The simulation results and the analytical expression have good

agreement for low values of the SNR. As the values increase, the number of errors decreases to only 2 errors in  $10^5$  bits at 9.2 dB SNR. With such small error counts the deviation of the Monte Carlo result from the exact value should be larger and this behaviour is clear in the data.

The data clearly shows a tradeoff between bit error rate, BER, and the transmitted signal power. With a given channel, the only way to increase the SNR is by boosting the transmitter power.

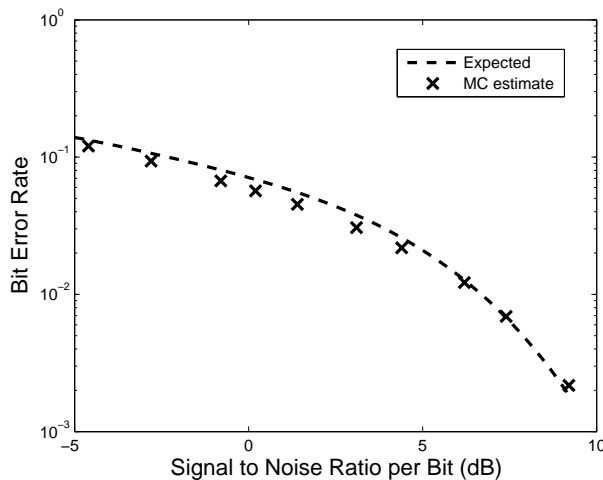


Figure 12: Monte Carlo results for the BER as a function of SNR for the 16 QAM system

## 4.2 4 QAM Analysis

The 4-QAM results are quite interesting. As figure 13 clearly shows, the BER of the 4-QAM system is far less than the 16-QAM system for a given signal to noise ratio. This behaviour is also expected as an error in 4-QAM requires the detector getting an incorrect phase by more than  $\pi/4$ . Due to the detector implementation, the demodulation only requires that the right quadrant of the constellation be picked.

In addition, the integration in the demodulator naturally causes the noise to cancel out as  $E[n(t)] = 0$  for white noise. The 16-QAM system has to deal

with errors in both amplitude and phase and thus is much more prone to error.

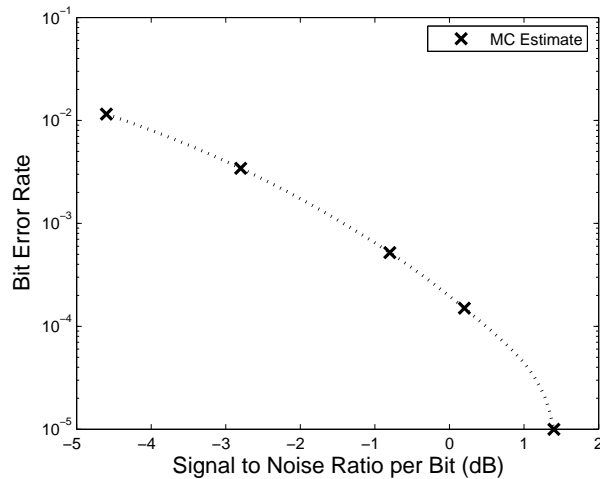


Figure 13: Monte Carlo results for the BER as a function of SNR for the 4 QAM system

## 5 Future Work

With a simulink implementation, the next step is to actually implement a communication system with an actual RF link. The communication system described in this report could be implemented in software on a device similar to the GNU Radio [1] with the channel being implemented as a physical link between a transmitter block and a receiver block. This “hardware” implementation would allow verification of the Monte Carlo results given above.

## 6 Conclusion

This report has detailed our implementation of the complete Quadrature Amplitude Modulation scheme as a software simulation. At each stage, care was taken to implement the best available method and we have also analyzed the practical implications of the chosen method. Due to the complexity of the design, we had to resort to using the Monte Carlo method which allowed us to estimate the performance of the system based on repetitive experiments. Our results match the theoretical values exactly. As always in Engineering, a balance between higher throughput at the cost of bit error rate must be met. This is a task that must be determined by the design constraints and functionality requirements.

## References

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