Feedback control of subcritical instabilities
PHYS484 Project Proposal

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1 Motivation

The Complex Ginzburg-Landau equation (CGLE),

$$A_t = A + (1 + i\alpha)\nabla^2 A + (1 + i\beta)|A|^2 A$$

is a rich source of interesting phenomena. The parameters α and β cause the equation to show variations from the purely relaxational[1] GLE (for the $\alpha,\beta = 0$ limit) to the nonlinear Schrodinger equation[2] (for the $\alpha,\beta \to \infty$ limit.) In fact, the CGLE is often studied as a prototype equation for spatiotemporal chaos.

[1] the real GLE can be derived from a Lyapunov functional, which is relaxational under the dynamics of the system

[2] soliton solutions are well known

2 Background

Amplitude equations describe slow modulation in space and time near the threshold for an instability. If the instability is supercritical, the dynamics is described by the CGLE, as extensively studied in [1]. On the other hand, subcritical instabilities cannot usually be described by the CGLE.

A recent paper[2] demonstrates that the weakly non-linear blowup of the subcritical CGLE can be controlled by means of a global feedback. This is going to be the focus of the project.

3 Analytics & Numerics

My specific plans for the project is as the following. I intend to work through the derivation of traveling wave solutions to the CGLE as presented in the primary reference[2]. This is similar to the phase winding solutions we derived in class, and the paper[5]. Next, a similar analysis needs to be done for the pulse solution because the entirety of the paper deals with this kind of solution. The authors say that the derivation is similar to the Nozaki-Bekki solution of a supercritical CGLE[3].

The paper does numerical simulations of the 1D and 2D subcritical CGLE and demonstrates stable dynamics. This is done using a pseudospectral method with periodic boundary conditions. I intend to reproduce their results.

4 Applications

Thus far, the project hasn't been motivated by a physical application, but this shouldn't be hard. The CGLE appears in diverse contexts, e.g., Rayleigh-Benard convection, Maragoni convection, contact line stability in thin liquid films, chemical oscillations, multi-mode lasers, amongst others (see [2] for references.) The application is yet to be determined.

References

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